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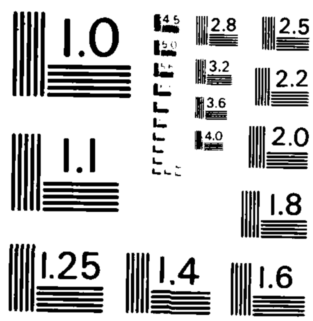

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IMPLEMENTATION AND COMPARISON OF MULTIPLE SOURCE LOCATION ALGORITHMS: RESULTS OF A SIMULATION

BY R. S. HEBBERT, L. T. BARKAKATI

UNDERWATER SYSTEMS DEPARTMENT

1 SEPTEMBER 1983

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC/TR 83-12	2. GOVT ACCESSION NO. AD-A146 241	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) IMPLEMENTATION AND COMPARISON OF MULTIPLE SOURCE LOCATION ALGORITHMS: RESULTS OF A SIMULATION		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. S. HEBBERT, L. T. BARKAKATI		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS NAVAL SURFACE WEAPONS CENTER (Code U22) WHITE OAK, SILVER SPRING, MD 20910		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Started under 6.2 Block
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 1 September 1983
		13. NUMBER OF PAGES 32
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

LINEAR ARRAY
CLASSIFICAL METHOD
ADAPTIVE METHOD
MAXIMUM ENTROPY METHOD
EIGENVECTOR METHOD

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents the results of a study of several source location algorithms, viz., Classical, Adaptive, Maximum Entropy, and Eigenvector. The algorithms use signals received by a linear array of sensors to compute source bearings. The resolution and the detection threshold of the algorithms are compared. It is concluded that the performance of the eigenvector method is superior to that of the others. However, the eigenvector method is rather expensive in terms of computation effort. An approximate maximum

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entropy method was examined and found to have good resolution and low computational cost.

FOREWORD

This report compares the performance of several source location algorithms: Classical, Adaptive, Maximum Entropy, and Eigenvector. The results should be useful to those processing sonar signals from uniform line arrays.

Approved by:

A. M. Jox

A. M. JOX, Head
Surface ASW Division

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CHAPTER 1

INTRODUCTION

This report compares the resolution and the detection threshold of various multiple source location algorithms. Such algorithms are digital signal processing techniques that use signals received by an array of sensors and provide estimates of the bearings of the signal sources. We will confine our attention to linear arrays consisting of omni-directional acoustic sensors. Further we will assume the signals to be narrowband.

The methods under consideration are:

1) Classical method
2) Adaptive method
3) Maximum entropy method (approximate)
4) Eigenvector method

The first three methods involve estimating the signal power incident on the array from various bearings. The second and third method involve computation of the maximum-likelihood and maximum-entropy estimates of the power respectively (reference 1). The eigenvector method is based on certain properties of the eigenvectors of the correlation matrix of an array illuminated by a field of discrete sources (references 2 and 3). It represents the acoustic field as a finite sum of plane waves.

We will describe these methods in Chapter 2. The algorithms have been implemented by computer programs written in FORTRAN and have been tested with simulated data on the VAX-11/780 system at the Naval Surface Weapons Center, White Oak, Maryland. The specific details of the computer programs and the results of the simulation are described in Chapters 3 and 4 respectively. In Chapter 4 we also compare the resolution, detection threshold, and computaional effort of the four methods.

CHAPTER 2

DESCRIPTION OF THE METHODS

Before proceeding to describe the multiple source location algorithms, let us formulate the problem under consideration.

Consider a linear array with N hydrophones. The array responds to signals of frequency, f, i.e., it is a narrowband array with center frequency, f. There are M signal sources located at bearings angles θ_1, θ_2, ..., θ_M with respect to the array axis (Figure 1) and signal powers p_1, p_2, ..., p_M. The sources are assumed to be uncorrelated and the incident waves are planewaves. The noise at the hydrophones is assumed to be isotropic in the horizontal plane that contains the array sensors.

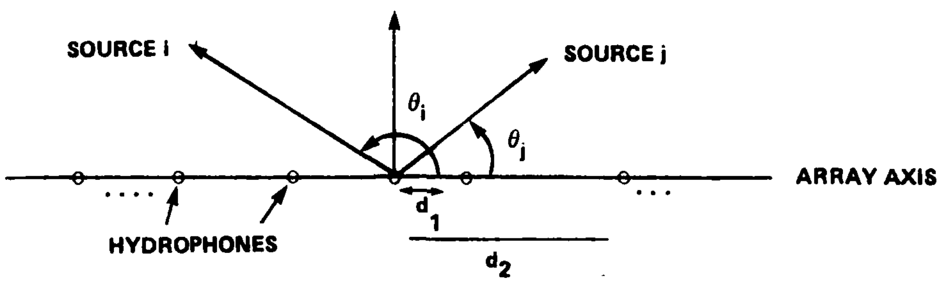

FIGURE 1. GEOMETRY OF THE PROBLEM

Let $V(\theta_i)$ denote the steering vector corresponding to the i-th source of planewave. Then $V(\theta_i)$ is given by:

$$V(\theta_i) = \begin{bmatrix} 1 \\ \exp(j2\pi d_1 \cos\theta_i/\lambda) \\ \exp(j2\pi d_2 \cos\theta_i/\lambda) \\ \vdots \\ \exp(j2\pi d_{N-1} \cos\theta_i/\lambda) \end{bmatrix} \tag{1}$$

where λ is the wavelength of the incident wave and $0, d_1, d_2, \ldots, d_{N-1}$ denote the hydrophone positions. We denote by Q_N the NxN covariance matrix of the noise signals received at the hydrophones. The covariance matrix of the total signal received at the array elements is denoted by Q. Since the sources are uncorrelated, we can write Q as

$$Q = Q_N + \sum_{i=1}^{M} p_i \, V(\theta_i) \, V^* (\theta_i) \qquad (2)$$

where * denotes complex conjugate tranpose.

The problem of interest to us can now be stated as follows: given Q, compute θ_i, i=1,2,...,M, the bearing of the sources. We assume that $M < N-1$.

Given the several existing methods of solving this problem, we wish to compare the resolution and the detection threshold of these methods. The resolution of a method tells us how close two sources can be before they become indistinguishable. The detection threshold indicates the signal power level above which the source can be located.

We will describe four different methods of estimating source locations in the rest of this chapter.

CLASSICAL METHOD

The estimate of power incident on the array from the bearing is taken to be

$$P(\theta) = V^*(\theta) Q V(\theta) \qquad (3)$$

The sources can be identified from a plot of $P(\theta)$ against θ, $\theta \in [0°, 180°]$. A typical plot for two distinct sources is shown in Figure 2. The sources are located at 45° and 120°.

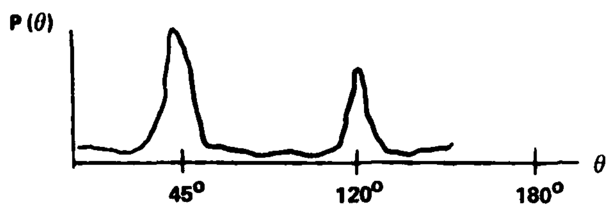

FIGURE 2. A PLOT OF TWO DISTINCT SOURCES

ADAPTIVE METHOD

In this method we use the maximum-likelihood estimate of the incident power distribution. This estimate is obtained by minimizing the mean square error for unit amplitude planewave signals arriving at the bearing for which the incident power is desired (reference 1). The adaptive power estimate versus bearing is given by

$$P(\theta) = \frac{1}{V^*(\theta)\ Q^{-1} V(\theta)} \tag{4}$$

A plot of $P(\theta)$ vs. θ should reveal the source locations. In our study, we found that the power estimate as given in Equation (4) often fails. Some modifications are necessary to make this method work. These are discussed in Chapter 3.

MAXIMUM ENTROPY METHOD

The power estimate used in this method is computed using the criterion that the estimate must be most random or have the maximum entropy of any estimate which is consistent with the measured covariance, Q (reference 1). The power distribution, $P(\theta)$, is given by

$$P(\theta) = \frac{1}{V^*(\theta)\ \Lambda\ V(\theta)} \tag{5}$$

where Λ is obtained from the self-consistency condition.

$$\frac{1}{\pi} \int_0^{\pi} \frac{V(\theta)V^*(\theta)}{V^*(\theta)\ \Lambda\ V(\theta)}\, d\theta = Q \tag{6}$$

In general no method is known for solving Equation (6) for Λ. A technique due to Burg (reference 1) *can be used to solve for Λ from* Equation (6) under the following conditions. The linear array consists of equally spaced hydrophones with spacing $d=0.5\lambda$. The covariance matrix Q is Toeplitz (we can make Q Toeplitz by replacing each element in a sub-diagonal by the average of all elements in that sub-diagonal). In this case, we can write Λ as

$$\Lambda = \gamma\gamma^* \tag{7}$$

Then it can be shown that reference 1

$$\Gamma = Q^{-1} c \tag{8}$$

where

$$c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{9}$$

We should note that Λ can be expressed in the form given by Equation (7) whenever the line array is composed of equally spaced sensors. For an equispaced line array with $d > 0.5\lambda$ we get ambiguous estimates for the source bearing.

There are no known results for the case $d < 0.5\lambda$. We can, however, use an approximate method in this case. This method is described in Chapter 3.

EIGENVECTOR METHOD

Here we determine the source locations from certain properties of the eigenvectors of the covariance matrix, Q, (references 2 and 3). To start with, suppose the noise signals at the array elements are uncorrelated with identical variance, σ^2. Then $Q_N = \sigma^2 I$, where I is an NxN identity matrix. Then;

$$Q_N = \sigma^2 I + \sum_{i=1}^{M} P_i \, V(\theta_i) V^*(\theta_i) \tag{10}$$

The algorithm is based on the following theorem due to C--toni and Godara (reference 3).

Theorem: Let W be the eigenvector of Q corresponding tc -e smallest eigenvalue. Then if $M \leqslant N-1$

$$W^* V(\theta_i) = 0, \quad i=1,\, 2, \ldots, M \tag{11}$$

We note that this theorem can be applied if the sources are correlated. In any case, the theorem suggests an algorithm for finding the source locations from the covariance matrix Q. The algorithm is as follows:

1) Find the eigenvalues and eigenvectors of Q

2) Find W, the eigenvector of Q corresponding to the smallest eigenvalue.

3) Solve for the roots of

$$W^* \begin{bmatrix} 1 \\ \exp(j2\pi d_1 \cos\theta/\lambda) \\ \exp(j2\pi d_2 \cos\theta/\lambda) \\ \vdots \\ \exp(j2\ d_{N-1} \cos\theta/\lambda) \end{bmatrix} = 0 \tag{12}$$

These roots give the source bearings,

4) Put the $V(\theta_i)$ into Equation (10) and find power, P_i, of each source. Then those sources corresponding to power greater than some

threshold value can be considered to be actually present.

The above algorithm is based on the assumption that the noise signals are uncorrelated (i.e., Q is proportional to I). We can use the same algorithm for correlated noise signals provided we prewhiten the covariance matrix, Q, and modify the eigenvector, W, before solving Equation (12). These modifications are indicated below:

1) Since Q is Hermitian, we can write it as
$Q = LL^*$

2) Modify Q by transforming it as follows:
$Q \leftarrow L\ Q(L\ )^*$

3) Find W, the eigenvector corresponding to the eigenvalue of the transformed Q. Now transform W as follows:

$$W^* \leftarrow W^*L$$

and use this W* in Equation (12).

The implemented version of this algorithm turned out to be numerically unstable (Chapter 3). A modified algorithm (reference 4) was used in the final implementation. We will describe this in the following chapter.

CHAPTER 3

IMPLEMENTATION OF THE ALGORITHMS

In Chapter 2 we have described four methods of estimating source bearings using a linear hydrophone array. In our simulation we allowed the sensor positions to be perturbed by a small Gaussian random displacement from their nominal positions. We use spacings of $.3\lambda$ (λ is the wave length of the incident wave). This in turn gives us a rather singular covariance matrix. Modifications are therefore needed for implementation of the algorithms.

MODIFICATIONS TO THE CLASSICAL METHOD

Classical method has poor resolution and detection threshold. To improve the detection threshold we do the following:

1) estimate s (power of the background noise)

2) compute $R = Q - sQ_N$

3) then $P(\theta) = V^*(\theta)RV(\theta)$

MODIFICATIONS TO THE ADAPTIVE METHOD

When implemented using Equation (4), the power distribution versus bearing, $P(\theta)$, given by the adaptive method is erroneous. This is due to numerical instability in inverting the covariance matrix, Q. Q is ill-conditioned in case of small spacings (over sampled), i.e., the spread between its largest eigenvalue and its smallest one is very large. The inevitable uncertainties in the hydrophone locations produce large percentage changes in the smaller eigenvalues of Q. This in turn affects the evaluation of $P(\theta)$. Our experience has shown that the degrading effects of this problem can be alleviated if we modify the diagonal elements of Q as follows:

$$q_{ii} \leftarrow q_{ii} + 0.01\, q_{11}$$

MODIFICATIONS TO THE MAXIMUM ENTROPY METHOD

For an equally spaced array with spacing $d < 0.5\lambda$ and for a Toeplitz covariance matrix we use the following method to find the source bearings.

Let Λ be defined by Equation (7). Then it can be shown that (see Appendix A)

$$\Gamma \approx Q^{-1}\eta \tag{13}$$

where

$$\eta = \begin{bmatrix} J_0(0) \\ J_0(\beta) \\ \cdot \\ \cdot \\ \cdot \\ J_0((N-1)\beta) \end{bmatrix} \tag{14}$$

$$\beta = \frac{2\pi d}{\lambda} \tag{15}$$

and J_0 is the zeroth order Bessel function of the first kind.

The maximum entropy estimate of the source bearings is then given by the angles where the local minima of $|V^*(\theta)\Gamma|$ occur.

MODIFICATIONS TO THE EIGENVECTOR METHOD

The eigenvector method in the modified form is as follows: Let Q_N denote the covariance matrix of an assumed noise model, then

$$Q = sQ_N + \sum_{i=1}^{M} P_i V(\theta_i)V^*(\theta_i) \tag{16}$$

where s indicates the noise level. Let Q be Toeplitz. The following algorithm gives us an estimate of the source bearings.

1) Find $\hat{s}$, an estimate of s and write

$$R = Q - \hat{s}Q_N \tag{17}$$

2) Find eigenvectors and eigenvalues of R

3) Estimate M, the number of sources present. This can be done by studying the eigenvalues of R.

4) Find the source bearings by solving:

$$\min_{\theta} \; |\gamma^* V(\theta)| \tag{18}$$

where γ is an Nx(N-M) matrix formed by (N-M) eigenvectors of R corresponding to its (N-M) smallest eigenvalues (reference 4).

DETAILS OF IMPLEMENTATION

The source location algorithms are tested using simulated data. The usual approach is to generate sequences of data vectors for the linear array and then compute the covariance matrix Q from:

$$Q = \frac{1}{K} \sum_{i=1}^{K} X(i)X^*(i) \tag{19}$$

where X(i) denotes the i-th data vector. In the simulation, we have opted to generate Q directly. Our method incorporates the effects of computing Q using Equation (19) with finite K. (See Appendix B.)

The noise covariance matrix, Q_N, is generated by

$$Q_N = \frac{1}{N} \sum_{i=1}^{k} V(\theta_i)V^*(\theta_i) \tag{20}$$

where $V(\theta_i)$ is given by Equation (1) with $\theta_i = \frac{\pi}{k}(i-0.5)$. A deterministic Q is then computed from Equation (2), with P_i and θ_i, i=1,2,...,M appropriate for the simulation being made. In order to introduce the statistical effects of computing Q using Equation (19) with finite K, we use the following algorithm.

1) factor Q as : Q = UU* where U is the Choleski factor of Q

2) generate a perturbed identity matrix, $\tilde{I}$, (see Appendix B)

3) Define Q as : $Q = U\tilde{I}U^*$

4) To smooth Q we average all duplicated inter-element correlations in the covariance matrix. In the special case of equal spacing this involves averaging the elements down each diagonal resulting in a Toeplitz matrix. This is necessary to obtain a covariance estimate consistent with the model we use.

CHAPTER 4

RESULTS OF THE SIMULATION

In order to compare the performance of the four algorithms, we generate a sequence of covariance matrices corresponding to a number of time cuts. These covariance matrices contain a target track together with background noise. In generating these covariance matrices we assume that the position of each array sensor has been perturbed by a small Gaussian random displacement (σ =.01 on both coordinates). In our simulation we use a linear array with 8 sensors of spacings equal to $.3\gamma$. Each of the implemented algorithms is applied to this data and the estimated target track is plotted. The detection threshold and the resolution of an algorithm is found from the target track computed by that algorithm. We vary the signal power level to study the resolution. We also vary the number of post integrations (i.e., the number of averaging) and study their effect on the resolution and detection threshold of the algorithm.

DETECTION THRESHOLD

The detection threshold is defined as the signal power level above which the source can be located. In order to compare the detection threshold of the algorithms it is necessary to calculate, for each method, another threshold which we will call the "decision threshold". An algorithm would indicate the presence of a source at a certain bearing if the power from that direction exceeds the "decision threshold" of that algorithm.

The decision threshold of each algorithm is found in the following manner. We generate a large number ($\approx$1,000) of covariance matrices containing noise only. For each covariance matrix, an algorithm would compute the peak power. The maximum in the sequence of peak power is used as the decision threshold of the algorithm for a false alarm rate of approximately .001.

To find the detection threshold of an algorithm, we generate a sequence of covariance matrices containing a sawtooth track (see Figure 3). At intervals of 20 time cuts the signal power level is decreased by 1 dB. An algorithm is then applied to the sequence of covariance matrices. At each time cut, the algorithm produces a source location provided the power received from that direction exceeds the decision threshold of that algorithm. The detection

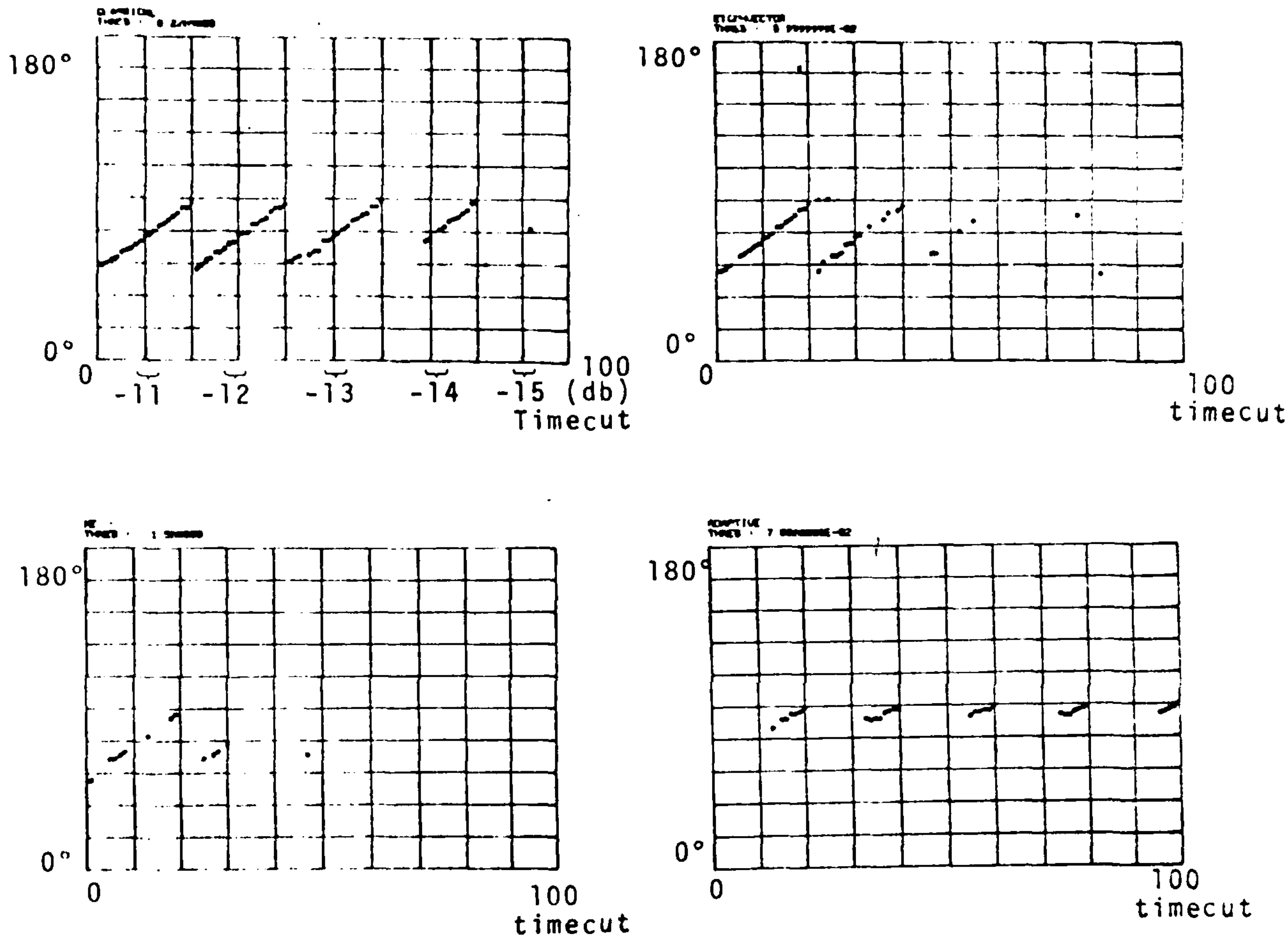

FIGURE 3. SAWTOOTH TRACKS FOR ESTIMATING DETECTION THRESHOLD OF ALGORITHMS

threshold is given by the signal power level at which the algoritnm finds the source approximately fifty percent of the time.

TABLE 1. COMPARISON OF CLASSICAL, ADAPTIVE, MAXIMUM ENTROPY, EIGENVECTOR FOR DETECTION THRESHOLD

# of post Integrations	Classical	Adaptive	M.E.	Eigenvector
1,000	-13.5 dB	More than -11 dB	-11 dB	-11.5 dB
100	- 8.5 dB	More than - 7 dB	- 7 dB	- 7.5 dB

Table 1 indicates that the classical method performs better than the others in detecting a single signal. The adaptive method has poor detection threshold, and it gives a biased result. We also learned that the perturbation of sensor positions does not change performance significantly. However, the variance of estimated bearings increases due to perturbation of array sensor positions.

RESOLUTION

The resolution of each algorithm is found as follows: a sequence of covariance matrices is generated which contain an interference line and a track approaching this line. The separation at which the two sources become barely distinguishable gives the resolution of the algorithm. At each time cut, a reasonable quantitative measure of the resolvability would be minimum angle separation above which two signals are resolved and below which they are not (i.e., minimum angle between two signals such that they resolve in the sense that the estimated power spectra display two distinct peaks for classical, adaptive and maximum entropy algorithm).

The resolution of the classical method can be estimated as follows: Suppose we have two signals, one located at x_0 and the other at $-x_0$. Let $V(x)$ be the steering vector at x where $x=\cos(\theta)$ (i.e., the k-th element of $V(x)= \exp(i2\pi sxk/\lambda)$ where s is the separation between array sensors and λ denotes the wavelength of the incident wave).

Then the power spectrum at x, $\sigma(x)$, is given by

$$\sigma(x) \quad V^*(x)\,|V(x_o)V^*(x_o) + V(-x_o)\,|\,V(x)$$

from which we can write

$$\sigma(o) = 2\left[\frac{\sin(2\pi s x_o N/\lambda)}{\sin(\pi s x_o/\lambda)}\right]^2$$

where N is the number of array sensors and

$$\sigma(x_o) = N^2 + \left[\frac{\sin(2\pi s x_o N/\lambda)}{\sin(2\pi s x_o/\lambda)}\right]^2$$

Let $\beta = \pi s x_o N/\lambda$

then $\frac{1}{N^2}\,\sigma(o) \approx 2(\sin\beta/\beta)^2$ (since $\sin(\pi s x_o/\lambda) \approx \beta$ for $N >> 1$)

and $\frac{1}{N}\,(x_o) \approx 1 + [\sin 2\beta/2\beta]^2$

therefore $\dfrac{\sigma(x_o)}{\sigma(o)} = \dfrac{4\beta^2 + (\sin 2\beta)^2}{8\sin^2\beta} = \dfrac{1}{2}\left[\left(\dfrac{\beta}{\sin\beta}\right)^2 + \cos^2\beta\right]$

We have resolution when $\sigma(x_o)/\sigma(1) > 1$ or minimum resolution occurs when $\sigma(x_o)/\sigma(o) = 1$, in this case, we get: $\beta = .9\frac{\pi}{2}$. Minimum resolution for classical $\approx$ 21° for N=8, s/λ = .3.

To study the effect of the signal power on the resolution, we have investigated a number of simulations using different signal power. The main result which we have expected and observed is that the stronger the signal power the better the resolution (with the exception of the classical algorithm).

TABLE 2. COMPARISON OF CLASSICAL, ADAPTIVE, MAXIMUM ENTROPY, EIGENVECTOR FOR RESOLUTION USING 1000 POST INTEGRATIONS (see Figure 4)

RESOLUTION OF

Signal Power	Classical	Adaptive	M.E.	Eigenvector
0 db	21°	15°	9°	4°
-6 db	21°	19°	12°	8°
-12 db	21°	23°	21°	11°

TABLE 3. COMPARISON OF CLASSICAL, ADAPTIVE, MAXIMUM ENTROPY, EIGENVECTOR FOR RESOLUTION USING 100 POST INTEGRATIONS

RESOLUTION OF

Signal Power	Classical	Adaptive	M.E.	Eigenvector
0 db	22°	15°	10°	7.5°
-6 db	22°	19°	12°	9.5°
-9 db	22°	20°	15°	12.0°

Tables 2 and 3 indicate that the eigenvector method performs consistently better than the others in resolving targets. Classical method has poor resolution, but the resolution does not depend on power in this case. Therefore in case of very weak signal the classical algorithm can be used successfully while the others are no longer applicable.

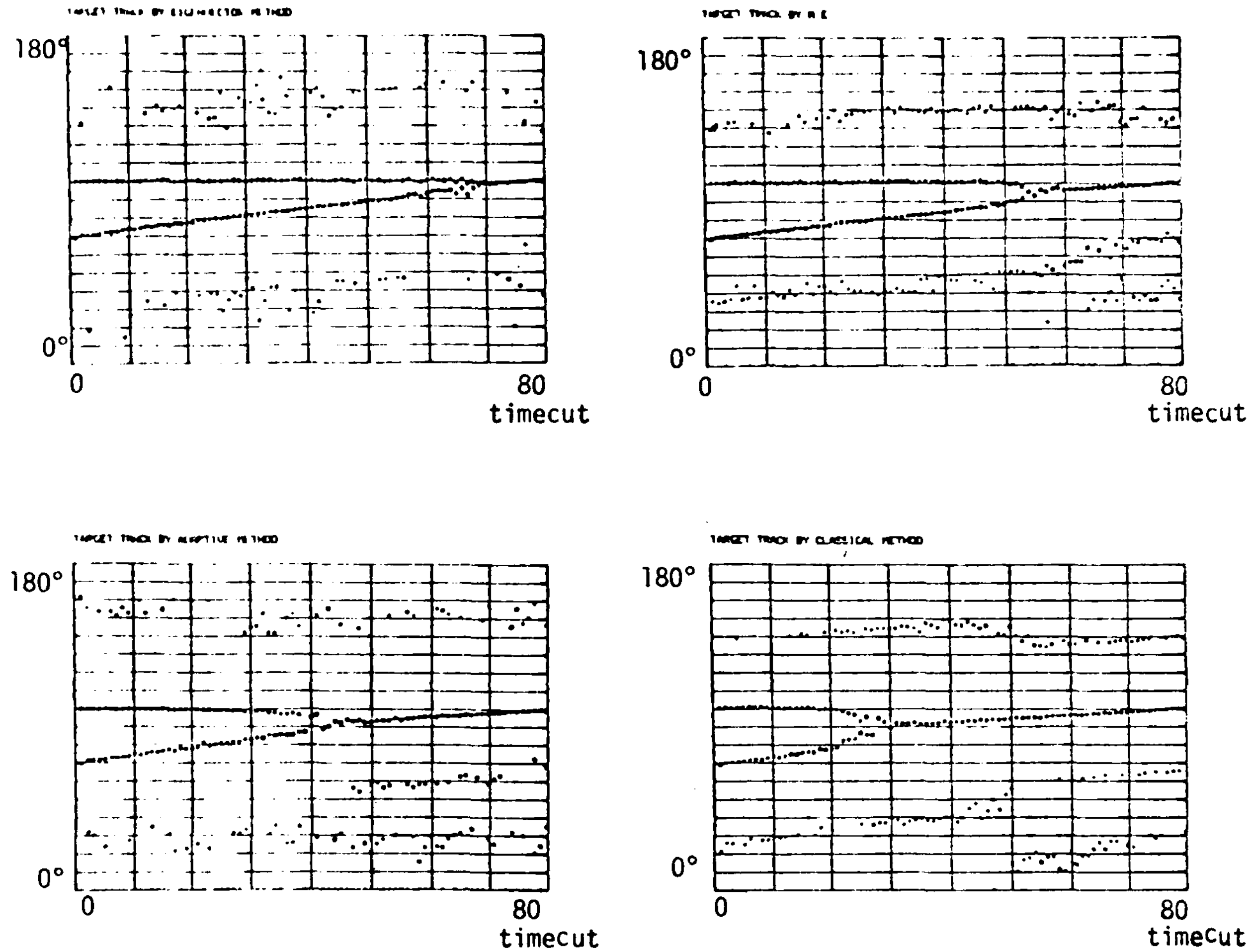

FIGURE 4. TARGET TRACKS FOR ESTIMATING RESOLUTION OF ALGORITHMS

COMPUTATION TIME

We have implemented the four source location algorithms, viz, classical, adaptive, maximum entropy and eigenvector algorithm. The programs were written in FORTRAN with no particular effort made to make them time and memory efficient. We will estimate (Table 4) the number of operations, in each algorithm, in flops (a "flop" is roughly equivalent to the amount of time required to perform one multiplication, one addition, and perhaps a few address calculations).

TABLE 4. COMPARISON OF CLASSICAL, ADAPTIVE, MAXIMUM ENTROPY, EIGENVECTOR FOR COMPUTATIONAL TIME (FLOPS)

Classical	Adaptive	M. E.	Eigenvector
15N	$15N+3N^2$	$15N+3N^2$	$15N+N^3$

N: number of array sensors.

In all fairness, we should make it clear that these estimates of number of computations necessary to implement each algorithm are by no means exact yet the numbers do indicate that the eigenvector algorithm takes much more time to execute than the other algorithms.

CHAPTER 5

CONCLUSIONS

The results of a study of source location algorithms have been presented. The algorithms were compared on the basis of their ability to resolve closely positioned sources and locate low power signals. The computational effort involved in implementing the algorithms was also considered.

The classical method has the best detection threshold, approximately 1 to 3 db less than that of the others. The eigenvector method comes next, followed by the maximum entropy method and lastly, the adaptive method. It was found that increasing the number of post integrations from 100 to 1,000, in spite of coordinate perturbations, reduces the detection threshold by nearly the expected amounts.

The eigenvector method provides the best resolution; the maximum entropy and the adaptive method follow with gradually worsening resolution. The classical method can resolve signals only when they are very far apart (i.e., 21° apart in the case of 8 sensors with 0.3λ spacings).

The classical method requires the least amount of computation and is very cost effective provided resolution is not important. The computational efforts involved in the adaptive and the maximum entropy methods are about the same. The eigenvector method is rather expensive compared to the other methods.

In our simulation an array of eight sensors was used. For arrays involving a large number of sensors, our conjecture is that the maximum entropy methods provides the best performance at reasonable computation cost. Further investigation is necessary to confirm this conjecture.

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APPENDIX A

DERIVATION OF APPROXIMATE ME METHOD

In this appendix we derive an approximation for the M.E. method. We also provide justifications for this approximation in simple cases.

We have:

$$Q = \int_{-\pi}^{\pi} d\theta\, V(\theta)\, V^* (\theta)\, \sigma(\theta) \tag{A.1}$$

where $\sigma(\theta)$ is the power density at bearing θ

$$V_k(\theta) = e^{i\beta\cos\theta k} \quad (k^{th} \text{ element of } V(\theta)), \text{ where } \beta = \frac{2\pi d}{\lambda}$$

then

$$Q_{kl} = \int_{-\pi}^{\pi} e^{i\beta(k-l)\cos\theta} \sigma(\theta)\, d\theta$$

Let $x = \beta\cos\theta$, we get:

$$Q_{kl} = \int_{-\beta}^{\beta} \frac{dx\sigma(x) \quad e^{ix(k-l)}}{\sqrt{\beta^2 - x^2}}$$

$$\text{Let } f(x) = \frac{1}{\sqrt{\beta^2 - x^2}} \qquad x^2 < \beta^2$$

$$= o \qquad x^2 > \beta^2$$

Express f(x) using Fourier series, we obtain

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

$$c_n = \int_{-\pi}^{\pi} \frac{dx}{2\pi} \frac{e^{-inx}}{\sqrt{1-\left(\frac{X}{\beta}\right)^2}} = \int_{-\pi}^{\pi} d\theta\, e^{-i\beta n\cos\theta}$$

$$c_n = J_o(\beta n)$$

Then

$$Q_{kl} = \sum_{n=-\infty}^{\infty} J_o(\beta n) \int_{-\pi}^{\pi} e^{ix(k-l+n)}\, \sigma(x)\, dx \tag{A.2}$$

From equations (6) and (7), we have

$$\sigma(0) = \frac{1}{V^*(0)\gamma\gamma^* V(0)}$$

$$Q_{kl} = \sum_{n=-\infty}^{\infty} J_o(\beta n) \int_{-\pi}^{\pi} \frac{e^{i(k-l+n)x} dx}{V^*(x)\gamma\gamma^* V(x)} \tag{A.3}$$

$$\text{Let } a_m = \int_{-\pi}^{\pi} \frac{dx\ e^{-imx}}{(V^*\gamma)(\gamma^* V)} \tag{A.4}$$

Then $a_{-m} = a_m{}^*$

$$Q_{kl} = \sum_{n=-\infty}^{\infty} J_o(\beta n)\ a_{l-k-n}$$

$$\sum_{l=0}^{N-1} Q_{kl}\gamma_l = \sum_{n=-\infty}^{\infty} J_o(\beta n) \sum_{l=0}^{N-1} \gamma_l a_{l-k-n} \tag{A.5}$$

$$\text{Let } \sigma_l = \sum_{i=0}^{N} \gamma_i a_{i+l}$$

It can be shown that

$$\sigma_l = \begin{cases} 0 & l<0 \\ \dfrac{1}{\gamma_0{}^*} & l=0 \end{cases}$$

For $l > 0$

$$\sigma_1 = \frac{(\gamma_1{}^*)^2}{(\gamma_0{}^*)^2}, \quad \sigma_2 = \frac{(\gamma_1{}^*)^2 - \gamma_0{}^*\ \gamma_2{}^*}{(\gamma_0{}^*)^2}, \quad \ldots$$

Therefore,

$$\sum_{l=0}^{N-1} Q_{kl}\gamma_l = \frac{1}{\gamma_0{}^*}\left[J_o(\beta k) - \left(\frac{\gamma_1}{\gamma_2}\right)^* J_o(\beta(k+1)) + \ldots\right]$$

Taking only the 1st term of the series and ignoring the constant $(1/\gamma_o^*)$ we get:

$$\gamma \approx Q^{-1} \begin{bmatrix} J_o(o) \\ J_o(\beta) \\ \cdot \\ \cdot \\ \cdot \\ J_o((N-1)\beta) \end{bmatrix} \qquad (A.6)$$

Let J be defined as:

$$J = \begin{bmatrix} J_o(o) \\ J_o(\beta) \\ J_o(2\beta) \\ \cdot \\ \cdot \\ \cdot \\ J_o((N-1)\beta) \end{bmatrix}$$

If the background noise is isotropic in the plane which contains the array sensors then J is the first column of the covariance of background noise. In other words,

$$J = Q_N e$$

where Q_N is the covariance of background noise and

$$e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Suppose we have no signal (i.e., $Q = Q_n$) then with the approximation in Equation (6), we get:

$$\gamma = Q^{-1} J = Q_N^{-1} Q_N e = e$$

Therefore $V^*(z)\,\gamma = z^{N-1}$

So with this approximation of M.E. we obtain a flat power spectral (i.e., all poles at the origin) as when there is no signal.

Suppose we now have one planewave, then:

$$Q = Q_N + pV(\theta_o)V^*(\theta_o)$$

$$Q^{-1} = Q_N^{-1} - \frac{P}{1+A_p} Q_N^{-1} V(\theta_o)V^*(\theta_o)\, Q_N^{-1}$$

where $A = V^*(\theta_o)\, Q_N^{-1} V(\theta_o)$

$$\gamma = Q^{-1} Q_N e$$

therefore:

$$\gamma = (Q_N^{-1} - \frac{p}{1+A_p}\; Q_N^{-1}\; V(\theta_o)V^*(\theta_o)\; Q_N^{-1})\; Q_N\; e$$

$$\gamma = e - \frac{p}{1+A_p}\; Q_N^{-1} V(\theta_o)V^*(\theta_o)\; e$$

$$= e - (\frac{p}{1+A_p})\; Q_N^{-1}\; V(\theta_o)$$

$$V^*(\theta)\; \alpha = 1 - (\frac{p}{1+A_p})V^*(\theta)\; Q_N^{-1} V(\theta_o)$$

The second term in the right hand side of the above equation is the optimum processing for one plane wave. So using this approximation we process the signal optimumly in case of one plane wave.

APPENDIX B

COVARIANCE MATRIX SIMULATION

In this Appendix we provide justifications for generating covariance matrix in our simulation.

Let y(k) be the k-th observation vector

Then:

$$Q = \frac{1}{M} \sum_{k=1}^{M} y(k)y^*(k)$$

$\overline{y(k)y^*(k)}$ can be factored as: $\overline{yy^*} = UU^*$

Let z(k) be such that $\overline{z(k)z(k)^*} = I$

Then y(k) can be written as:

$$Y(k) = Uz(k)$$

$$\sum_{k=1}^{M} y(k)y^*(k) = U\left[\sum_{k=1}^{M} z(k)z^*(k)\right] U^*$$

$$Q = \frac{1}{M} \sum_{k=1}^{M} y(k)y^*(k) = U \left[\frac{1}{M} \sum_{k=1}^{M} z(k)z^*(k)\right] U^* \qquad \text{(B.1)}$$

$$Q = U\hat{I}U^*$$

where $\hat{I} = \frac{1}{M} \sum_{k=1}^{M} z(k)z^*(k)$ (B.2)

So Equations (1) and (2) account for Steps (1), (2), and (3) (in Chapter 3) in generating covariance matrices with statistics.

To generate a perturbed identify matrix, $\hat{I}$, let us define L_M such that

$$L_M L_M^* = \sum_{k=1}^{M} z(k)z(k)^*$$

In other words, L_M is a Choleski factor of $\sum_{k=1}^{M} z(k)z(k)^*$

It can be shown that L_M is a lower Triangular matrix where diagonal elements are chi distributed, and off diagonal elements are Rayleigh distributed with unit variance. All the elements are independent.

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